m^2/sec; T, specimen temperature, K; R, specimen radius, m; b, radius of the heat flux on the exposure surface, m; q_0, heat flux density amplitude, W/m^2; ω, modulation frequency of the heat flux, rad/sec.

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INFLUENCE OF PARTICLE COMPACTION ON THERMOPHYSICAL PROPERTIES OF A LAYER

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Problems of compaction of a layer of deposited particles and its influence on the thermophysical parameters of the layer are considered.

Most technological processes, performed with disperse systems, are accompanied by deposition of the solid phase on the heat-and-mass-exchange surface, which substantially affects the coefficients of heat, mass, and momentum transfer [1, 2]. Deposition of particles of various kinds on the surface is determined by many factors, among which it is important to distinguish hydrodynamic and thermodynamic conditions, rheological properties of the disperse system, adhesive compatibility of particles with the surface in flow, physicochemical transformations in the boundary layer, etc. At the same time the layer formed by particle deposition is subject to deformation and compaction under the action of external deforming stresses, which substantially affects the heat transfer through a two-layer wall and the thermophysical properties of the deposited layer proper. The mechanism of compacting the layer of disperse particles may be classified in the following manner: a) the dislocation mechanism, whose essence consists of redislocation of particles in elastobound systems under the action of external loads; b) mechanical compaction due to the action of external deforming stresses and mass forces [3]; c) compaction by diffusion of small particles under the action of diffusion thermophoresis [4]; d) the capillary mechanism of compaction in the presence of phase transformations (melting, dissolution, etc.) [4].

One-dimensional compaction of particles in the layer under the action of the external deforming stresses σ_D may be represented by the equation of porosity variation in the form [3]

$$\frac{d\Theta}{dt} = -(1-\Theta)\,\eta_s^{-1}\,\sigma_D. \tag{1}$$

Expressing the bulk viscosity ξ_s in terms of the shear one and changing from the substantional derivative to the local ones, we obtain

$$\frac{\partial\Theta}{\partial t} + V_s \operatorname{grad}\Theta = -\frac{3}{4}\,\Theta\eta_s^{-1}\,\sigma_D, \tag{2}$$

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where V_s is the velocity vector of particles in the layer, which, as a first approximation, may be assumed equal to the rate of squeezing a liquid out of pores as a result of compaction.

We will consider a simplified hydrodynamic problem of squeezing the liquid out of a hypothetical cylindrical pore of radius R_p and length $x = \delta$ by compaction of the layer. The equations of continuity and distribution of velocity of the liquid phase for $U_r << U_x$, $\partial U_x/\partial x << \partial U_x/\partial r$ and in the quasistationary approximation have the form

$$\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_x}{\partial r}\right) = \frac{1}{\rho}\frac{\partial P}{\partial x}; \quad \frac{1}{r}\frac{\partial}{\partial r}(rU_r) + \frac{\partial U_x}{\partial x} = 0, \tag{3}$$

where U_r and U_x are the radial and axial components of the liquid velocity in the pore. With allowance made for the hydrodynamic slip on the macropore surface for the Knudsen numbers within $0.01 \le \mathrm{Kn} \le 0.1$, the boundary conditions will be represented in the form

$$\begin{gathered} r = R_p,\ U_r = -U_g,\ U_x = -k_{hs}\left.\frac{\partial U_x}{\partial r}\right|_{r=R_p}; \\ x = 0,\ U_x = U_r = 0,\ \frac{\partial P}{\partial x} = 0; \\ x = \delta,\ P = P_0, \end{gathered} \tag{4}$$

where U_g is the velocity of movement of the pore wall under the external load. Omitting the simplest transformations, we will present the solution (3) with the conditions (4) in the form

$$U_x = \frac{4U_g x}{R_p^2}\,\frac{R_p^2 - r^2 + 2k_{hs}R_p}{R_p + 4k_{hs}}, \tag{5}$$

where x is the statistical mean pore length, related to the thickness of the particle layer. Averaging the velocity U_x over the pore cross section, we obtain

$$U_{xs} = \frac{2\int\limits_0^{R_p} U_x r\,dr}{R_p^2} = \frac{U_g x}{R_p}. \tag{6}$$

Thus, as follows from (6), the rate of squeezing the liquid out of the pores increases as the external load increases and the pore radius decreases. In the general case the value for velocity of longitudinal travel of particles across the layer thickness depends on many factors and is described by the equation [5]

$$\frac{\partial V_s}{\partial q} = -\frac{3\sigma_d}{4\eta_1}\,\frac{1-\bar{\rho}}{\bar{\rho}^{m-1}}, \tag{7}$$

where q is the current mass in Lagrangian coordinates

$$q(x, t) = \int\limits_0^x \bar{\rho}(x, t)\,dx = \int\limits_0^x (1-\Theta)\,dx;$$

m is the index in the relations for viscosity; $\bar{\rho}$ is the density of the material with respect to the density of the incompressible base.

Assuming that at $q = 0$, $x = \delta$, $U_s(0) = -U_d$ and setting $m = 2$, from (7) we will obtain

$$V_s = U_d - \frac{3\sigma_d}{4\eta_s}\int\limits_0^x \Theta(x)\,dx. \tag{7a}$$

Substituting the given expression into (2), for one-dimensional compaction we write

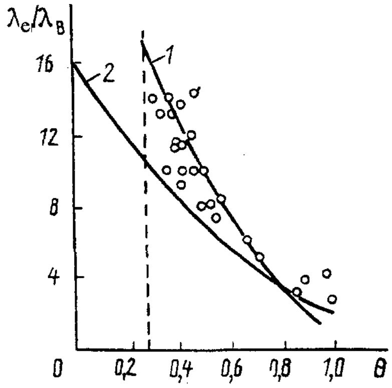

Fig. 1. Variation of the thermal conductivity coefficient of a particle layer: 1) formula (10); 2) (11).

$$\frac{\partial\Theta}{\partial t}+\left(U_d-\frac{3\sigma_d}{4\eta_s}\int_0^x\Theta(x)\,dx\right)\frac{\partial\Theta}{\partial x}=-\frac{3}{4}\Theta\eta_s^{-1}\sigma_d. \tag{8}$$

On introduction of the new function $\psi = \int\Theta(x)dx$, integrodifferential equation (8) is modified to the form

$$\frac{\partial^2\psi}{\partial t\partial x}+\left(U_d-\frac{3}{4}\eta_s^{-1}\sigma_d\psi\right)\frac{\partial^2\psi}{\partial x^2}=-\frac{3}{4}\eta_s^{-1}\sigma_d\frac{\partial\psi}{\partial x}.$$

The given quasilinear hyperbolic equation characterizes a variation of the layer porosity in thickness and in time as a result of its compaction. If we are to assume that the layer thickness is small, characteristic of the processes of particle deposition, then the weight of the upper layers of particles may be ignored. Then it can be assumed that $V_s \approx U_d$, and we will present the equation of compaction in the form

$$\frac{\partial\Theta}{\partial t}+U_d\frac{\partial\Theta}{\partial x}=-\frac{3}{4}\Theta\eta_s^{-1}\sigma_d;$$

$$\Theta(x,\ t)|_{t=0}=\Theta_0;\quad |\Theta(x,\ t)|<\infty.$$

We will represent the solution of the given equation as

$$\Theta=\Theta_0\exp\left[-\frac{3}{4}\eta_s^{-1}\sigma_d(t-x/U_d)\right]. \tag{9}$$

It is common knowledge that the coefficients of mass and heat transfer substantially depend on porosity. There are many formulas for calculating the effective thermal conductivity coefficient of a monodisperse layer of particles λ_e, among which it is pertinent to note the dependence [6]

$$\lambda_e=3\pi\lambda_B\ln\frac{0.43+0.31\Theta}{\Theta-0.26}, \tag{10}$$

which is adequate up to the organization of a dense ordered structure of identical spherical particles ($\Theta = 0.26$). As Fig. 1 shows, Eq. (10) more adequately describes the experimental data [7] than the equation [6]

$$\frac{\lambda_e}{\lambda_B}=\left(1-2\Theta\frac{1-\lambda_\Gamma/\lambda_B}{2+\lambda_\Gamma/\lambda_B}\right)\Big/\left(1+\Theta\frac{1-\lambda_\Gamma/\lambda_B}{2+\lambda_\Gamma/\lambda_B}\right). \tag{11}$$

Using (9), we will represent Eq. (10) as

$$\frac{\lambda_e}{\lambda_B} = 3\pi \ln \frac{0,43 + 0,31\Theta_0 \exp(-K(t - x/U_d)]}{\Theta_0 \exp(-K(t - x/U_d)) - 0,26}, \tag{12}$$

where $K = (3/4)\eta^{-1}{}_s\sigma_d$.

Equation (12) makes it possible to determine the distribution of the effective thermal conductivity coefficient in time and across the thickness of the layer in its compaction.

If we are to deal with the mean porosity over the layer thickness, then from (9) we obtain

$$\Theta = \Theta_0 \exp\left(-\frac{3}{4}\eta_s^{-1}\sigma_d t\right).$$

Since with small dimensions of pores the shear viscosity attains large values, $\eta_s{}^{-1} << 1$. Then we have

$$\Theta = \Theta_0 (1 - 0.75\eta_s^{-1}\sigma_d t);$$

$$\frac{\lambda_e}{\lambda_B} = 3\pi \ln \frac{0.43 + 0.31\Theta_0 (1 - 0.75\xi_s^{-1}\sigma_d t)}{\Theta_0 (1 - 0.75\xi_s^{-1}\sigma_d t) - 0.26}.$$

Thus, as the particle layer is compacted, from (10) and (11) it can be assumed that the thermal conductivity coefficient grows and in the limiting case is closer to the value of thermal conductivity of the particles. We will determine the density of the deposited layer of particles in the form

$$\rho(x, t) = \rho_m(1 - \Theta) = \rho_m[1 - \Theta_0 \exp(-K(t - x/U_d))],$$

whence it follows that in the hypothetical case $t \to \infty$, we have $\lim\limits_{t\to\infty} \rho(x, t) \to \rho_m$.

In conclusion we will point out that with large thicknesses of the layer of particles a substantial role in their compaction can be played by mass forces caused by the weight of the upper layers. Besides, in compaction of polydisperse particles, their dimensions or fluctuations of distribution in particle dimensions and the problems of their aggregative stability defined by the formation of coagulative structures will be of importance.

NOTATION

k_{hs}, coefficient of hydraulic slip; P, pressure; R_p, pore radius; U, velocity of liquid in pores; x, coordinate; t, deposited layer thickness; Θ, porosity; η_s, bulk viscosity; ν, viscosity of liquid in pores; ρ_m, density of material of particles; λ_B, thermal conductivity coefficient of particles.

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